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Crack growth of multiple-site damage: experimental and the extended finite element method prediction

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1. Introduction

The phenomenon of multiple site damage (MSD) is defined as that of multiple cracks of arbitrary lengths emanating from a row of fastener holes in a bonds, riveted lap joint in a pressurized fuselage of airplane [1,2]. Accurate prediction of fatigue crack growth of MSD is essential to assess the structural integrity of aging aircraft.

In this paper, a method, called extended finite element method (XFEM), is presented to study the fatigue growth of MSD cracks. The key ingredient in this method is that it permits the cracks to cut elements by introducing local enrichment functions in the displacement approximation to represent the discontinuous displacement across the crack lines [3-5].

In addition, the tests are performed on the fatigue behavior of aluminium 2024-T3 in order to determine the time of occurrence and crack growth path of fatigue damage. These test data are used to assess the numerical prediction based on the XFEM.

In the next section, we briefly present the displacement field enriched to model the discontinuities in a multiple crack problem. From which, the discretization of the equilibrium equations is derived. We also explain steps of fatigue crack growth analysis by the Paris' law in this section. Experimental work is concisely described in section 3. In section 4, the numerical simulation for testing models is done to illustrate the accuracy and efficiency of the XFEM in MSD problems. The conclusions are drawn in section 5.

2. XFEM crack modeling

2.1 The approximation of the displacement field

The XFEM treats the discontinuous displacement field across the cracks by the enriched functions. The finite element approximation $\boldsymbol{u}^h$ of the crack domain is given by [4,5]

$$\boldsymbol{u}^h = \sum_{I \in N} N_I \boldsymbol{u}_I^0 + \sum_{J \in E} \sum_{I \in N_{enr}^J} N_I \left(\Psi^J - \Psi_I^J \right) \boldsymbol{a}_I^J \qquad (1)$$

in which N is the set of node, N_I is the shape function of the finite element method and $\boldsymbol{u}_I^0$ is the nodal displacement of node I. E is the set of enrichment types, N_{enr}^J is the set of nodes related to enrichment type J, $\boldsymbol{\Psi}^J$ and $\boldsymbol{\Psi}_I^J$ are the enrichment functions of type J and the one at node I, $\boldsymbol{a}_I^J$ are enrichment parameters.

As shown in [5], the step function and the solution of linear elastic fracture mechanics are used as enrichment ones for the displacement field of any element completely cut by a crack and containing a crack tip, respectively. They are given by

$$\Psi^J = \Psi^{step} = sign(f) \tag{2}$$

$$\Psi^J = \Psi^{tip} = \left\{ \begin{matrix} \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}, \\ \sqrt{r}\cos\frac{\theta}{2}\sin\theta, \sqrt{r}\sin\frac{\theta}{2}\sin\theta \end{matrix} \right\} \tag{3}$$

where f is the signed distance function measured with respect to the current crack considered, r is distance from the current crack tip and θ is the angle measured respect to the tangent at the crack tip.

The coalescence of two cracks occurs if one crack reaches the other during a step of propagation. When this occurs, the approaching crack tip is annihilated or killed and the junction of the two cracks can be implemented by the combination of two step functions [5].

2.2 Weak form and discretized equations

Consider a two-dimensional body Ω bounded by Γ, containing multiple cracks Γ_c

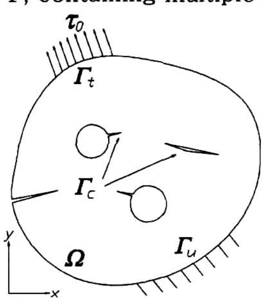

Fig. 1 A 2D-body containing multiple cracks

shown in Fig. 1. Prescribed displacements are imposed on Γ_u, while prescribed tractions τ_0 are imposed on Γ_t, where $\Gamma = \Gamma_u + \Gamma_t$. We assume that strains and displacements are small, the constitutive relation is given by Hooke' s law and the crack surfaces are free of traction. It is shown in reference [5] that the below weak form of equilibrium of the system is equivalent to the strong form, including the traction-free conditions on any lines of discontinuity:

$$\delta W^{\text{int}} = \delta W^{ext} \tag{4}$$

where

$$\delta W^{\text{int}} = \int_{\Omega \backslash \Gamma_c} \frac{\partial \delta \boldsymbol{u}}{\partial \boldsymbol{x}} \cdot \boldsymbol{\sigma} d\Omega \tag{5}$$

$$\delta W^{ext} = \int_{\Gamma_t} \delta \boldsymbol{u} \cdot \boldsymbol{\tau}_0 d\Gamma \tag{6}$$

Here, W^{int} and W^{ext}, respectively, are the internal and external work. $\delta \boldsymbol{u}$ is the test function (which vanishes along the displacement boundary Γ_u) and $\boldsymbol{\sigma}$ is the stress calculated from the trial displacement $\boldsymbol{u}$.

Use the approximation defined in (1) for both δu and u. From the weak form (4), one can obtain the discrete equilibrium equation:

$$f^{\text{int}} = f^{ext} \tag{7}$$

in which

$$f^{\text{int}} = \boldsymbol{Kq} = \int_{\Omega \backslash \Gamma_c} \boldsymbol{B}^T \boldsymbol{CB} d\Omega \boldsymbol{q} \tag{8}$$

$$f^{ext} = \int_{\Gamma_t} \boldsymbol{N}^T \tau_0 d\Gamma \tag{9}$$

Here, f^{int} and f^{ext} are the internal and external forces, respectively. K is the stiffness matrix, q is the generalized nodal displacements, B is the strain-displacement matrix and C is the constitutive matrix.

2.3 Fatigue crack growth

The SIFs are calculated by interaction integral [3,4]. The relation between crack growth and range of SIFs in the crack growth period under constant amplitude cyclic loading follows the Paris' law:

$$\frac{da}{dN} = C(\Delta K)^m \tag{10}$$

where a is the crack length, N is the number of cycles, C and m are the fatigue crack growth parameters. ΔK is defined as difference between maximum and minimum SIFs correspondent to the maximum and minimum applied loads, respectively.

To multiple-crack bodies, we choose a constant increment length Δa_{max} for the crack tip with the maximum SIF range ΔK_{max} and then at every crack tip the increment is given by:

$$\Delta a = \Delta a_{\max}\left(\frac{\Delta K}{\Delta K_{\max}}\right)^{m} \qquad (11)$$

The direction of the crack growth θ is determined based on the maximum hoop stress criterion [4,5]:

$$\theta = 2\arctan\frac{1}{4}\left(\rho_K \pm \sqrt{\rho_K^2 + 8}\right) \qquad (12)$$

where ρ_K is the ratio of the mode I to the mode II of SIFs.

3. Experimental work

This section is a synthesis of data obtained in the experimental work of ADD project. The specimens comprised two types of 2mm-thick aluminium alloy 2024-T3 sheets. One, so called FT-1018 and FT-1030, is a central crack specimen according to ASTM E 647 [6]. The other, named FT-2020 and FT-2033, is plate with some cracks emanating from holes similar to MSD cracks. The specimen geometry is presented in Fig. 2. The specimens were fatigue tested in an MTS with capacity of 100 kN.

The fatigue loads and number of cycles at failure are given in below table

Specimens	P_{max} (kN)	P_{min} (kN)	Cycles
FT-1018	11.2	2	6400
FT-1030	6.7	2	50500
FT-2020	10	2	5268
FT-2033	6	2	28500

4. XFEM prediction

A XFEM code is developed to simulate and predict crack propagation with the help of GiD software for pre-post processing.

Materials properties, Young's modulus and Poisson's ratio also obtained from experimental work with value 72600 MPa and 0.36, respectively.

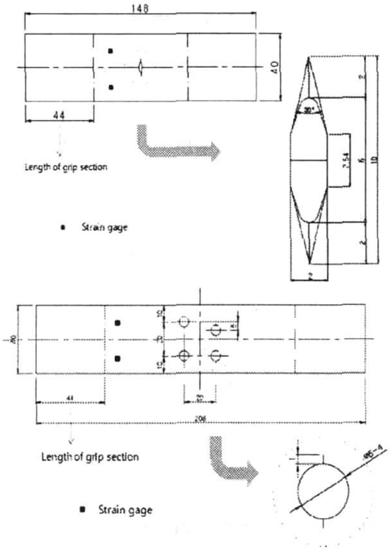

Fig. 2 The geometry of specimens

4.1 The central crack specimen, FT-1018, FT-1030

Data used for FT-1018 are C=2.5x10^{-10} (m/cycle MPa $m^{0.5}$) and m=2.78. For the case of FT-1030, C is similar but m=2.72. Fig. 3 compares the experimental results with the XFEM code prediction. With used fatigue parameters, the analysis is done well.

4.2 Specimens: FT-2020, FT-2033

Similarly, C parameter used for FT-2020 and FT-2033 is 2.5x10^{-10} (m/cycle MPa $m^{0.5}$). However, the values of m are little different because of dependence of stress ratio. Particularly, m=2.78 for FT-2020 and 2.89 for FT-2033.

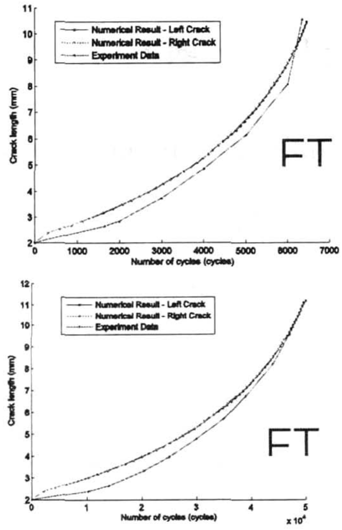

Fig. 3 Relation between crack length and number of cycles for central crack specimens

The number of cycles predicted by the numerical prediction is 5323 (or 1.04% in error) and 28272 (or 0.8% in error) for FT-2020 and FT-2033, respectively. Again, these results are in good agreement with the tested data in case of the used fatigue properties of material in range of reference [7].

Fig. 4 shows in similarity the crack paths at failure of the specimen FT-2020 and numerical simulation by the XFEM code.

5. Conclusions

The method, called XFEM, to simulate and predict fatigue crack growth of multiple crack problems has been presented. The paper also implements the coalescence of two cracks or as a crack reaches the boundary.

The XFEM code modeling and predicting the crack propagation of MSD problems is acceptable as verified by experimental data.

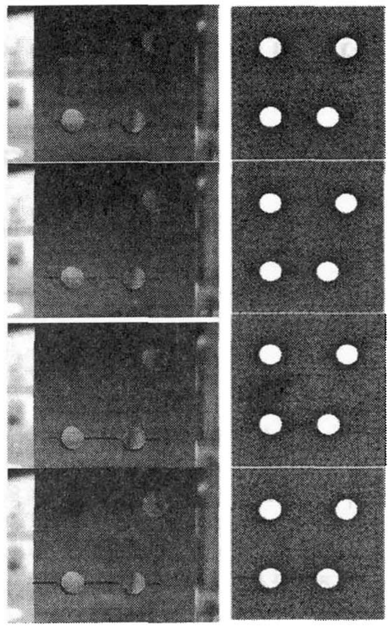

Fig. 4 Crack propagation profile under fatigue load of test (left) and the XFEM code (right)

Acknowledgments

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References

1. Park J, Atluri S. Fatigue growth of multiple-cracks near a row of fastener-holes in a fuselage lap-joint. Comp. Mech. 1993, 13: 189-03
2. Silva L, Goncalves J, Oliveira F, Castro P. Multiple-site damage in riveted lap-joints: experimental simulation and finite element prediction. Int. J. Fatigue 2000, 22: 319-38
3. Belytschko T, Black T. Elastic crack growth in finite elements with minimal remeshing. Int. J. Numer. Meth. Engng 1999, 45: 601-20
4. Moes N, Dolbow J, Belytschko T. A finite element method for crack growth without remeshing. Int. J. Numer. Meth. Engng. 1999, 46: 131-50
5. Zi G, Song J, Budyn E, Lee S, Belytschko T. A method for growing multiple cracks without remeshing and its application to fatigue crack growth. Modelling Simul. Mater. Sci. Eng. 2004, 12: 901-15
6. Annual book of ASTM Standards, 2002
7. Mann T. The influence of mean stress of fatigue crack propagation in aluminium alloys, Int. J. of Fatigue, 2007, 29: 1393-1401