CTOA와 점성균열모델에 기초한 잔류응력예측을 위한 확장유한요소법

Extended Finite Element Method for Residual Strength Prediction based on CTOA and Cohesive Crack Model

Chau-Dinh Thanh * · 김지환** · 유성문*** · 지광습****
Chau-Dinh, Thanh · Kim, Jihwan · Yu, Seongmun · Zi, Goangseup

1. Introduction

Residual strength prediction is essential to determine maximum load carrying capacity of a structure. It may be important to know not just the maximum load but behavior of the structure after this collapsed strength. To predict residual strength of such a structure a fracture criterion is needed to characterize stable crack growth. Numerous criteria have been proposed. Of these, the crack tip opening angle (CTOA) and cohesive crack model are the most suited for modeling crack extension in elastic plastic fracture process. A method, an extension of standard finite element methods, is employed because of its ability to simulate arbitrary crack propagation without remeshing. Based on the partition of unity properties, the method, namely XFEM, embeds local enrichment functions in displacement fields to model discontinuities. In this study, the XFEM is used to validate the stable crack growth in an aluminum sheet involving CTOA and cohesive crack model.

2. Discontinuous displacement field

For cracked elements discontinuous displacement field is decomposed into continuous and discontinuous parts. The discontinuous part is provided by enriched functions and additive degrees of freedom (Moes et al., 1999; Zi & Belytschko, 2003) as

$$\boldsymbol{u}^h = \sum_{I \in N} N_I \boldsymbol{u}_I^0 + \sum_{I \in N_{enr}} N_I \left(H - H_I\right) \boldsymbol{a}_I \tag{1}$$

in which N is the set of node, N_I is the shape function of the finite element method and u_I^0 is the nodal displacement of node I. N_{enr} is the set of enriched nodes, H and H_I are the sign functions and the one at node I, a_I are enrichment degrees of freedom. The sign function is defined $H(x) = \begin{cases} 1 & \textbf{\textit{if}}\ x > 0 \\ -1 & \textbf{\textit{if}}\ x < 0 \end{cases}$.

* Department of Civil, Environmental and Architectural Engineering, Korea University – E-mail: chdthanh@korea.ac.kr
** Department of Civil, Environmental and Architectural Engineering, Korea University – E-mail: jihwan@korea.ac.kr
*** Department of Civil, Environmental and Architectural Engineering, Korea University – E-mail: ysmgc@korea.ac.kr
**** Department of Civil, Environmental and Architectural Engineering, Korea University – E-mail: g-zi@korea.ac.kr

Only is the sign functions used here. This means we consider a crack that spans a complete element. The enriched functions vanish at the enriched nodes but not at integration points. This shifting reduces the enriched area and then the number of additive degrees of freedom.

The displacement jump across the crack is

$$[\![\boldsymbol{u}]\!] = 2\sum_{I \in N_{enr}} \boldsymbol{N}_I \boldsymbol{a}_I \tag{2}$$

3. Elastic plastic crack growth simulation

An initial crack in ductile material usually grows in a stable way under a monotonic increase of applied load. This is characterized by a permanent plastic zone around the crack tip during unloading. A fracture criterion is needed to predict the stable propagation during the fracture process. Two numerical models, namely a crack tip opening angle (CTOA) and a cohesive model, are briefly presented.

3.1 CTOA approach

Experimental studies in a number of metals show that the CTOA is constant after some initial crack growth and not depend on the specimen sizes. The CTOA value is defined as

$$\text{CTOA} = 2\tan^{-1}(\delta/2d) \tag{3}$$

where $\delta = \boldsymbol{n} \cdot [\![\boldsymbol{u}]\!]$ is the crack tip opening displacement measured at a specified distance d behind the crack tip and n is the normal direction of the crack.

There is still no unique definition or measuring standard for CTOA. In this paper, it is the angle measured by the crack tip and two points on the crack edges at a distance of 1.0mm behind the crack tip. The critical CTOA is 5.25 degrees provided by Dawicke and Newman based on experimental measurements (Chen et al., 1999).

3.2 Cohesive model approach

The cohesive crack model was introduced by Dugdal and Barenblatt. In the cohesive crack model, stress fields around the crack tip are governed by a traction-displacement relation across the crack faces. The cohesive stress depends on the crack opening displacement in the fracture process zone and reaches the maximum stress at the crack tip.

The weak form of equilibrium is given by

$$\delta W^{\text{int}} = \delta W^{ext} + \delta W^{coh} \tag{4}$$

Here, W^{int}, W^{ext} and W^{coh}, respectively, are the internal and external and cohesive work.

From (4) and (1) one can obtain the discrete equilibrium equation (Zi & Belytschko, 2003)

$$\boldsymbol{f}^{\text{int}} = \boldsymbol{f}^{ext} + \boldsymbol{f}^{coh} \tag{5}$$

in which

$$\boldsymbol{f}^{\text{int}} = \int_{\Omega \backslash \Gamma_c} \boldsymbol{B}^T \sigma d\Omega \boldsymbol{q} \qquad \boldsymbol{f}^{ext} = \int_{\Gamma_t} \boldsymbol{N}^T \tau_0 d\Gamma \qquad \boldsymbol{f}^{coh} = -2\int_{\Gamma_c} \tau^c \boldsymbol{N}^T \boldsymbol{n} d\Gamma \tag{6}$$

Here f^{int}, f^{ext} and f^{coh} are the internal, external and cohesive forces, respectively. N is the shape function matrix and B is the strain-displacement matrix. q is generalized nodal displacements including standard and additive degrees of freedom. σ is the stress. τ_0 is the external traction on Γ_t. τ^c is the cohesive traction along the cohesive crack Γ_c.

The cohesive law gives a relation between the cohesive traction τ^c and the crack opening displacement δ. In this study we only consider crack opening displacement normal to the crack line Γ_c. The normal cohesive traction relates to the normal crack opening displacement via the below equation

$$\tau^c(\delta) = \begin{cases} f_t\left(1 - \dfrac{\delta}{\delta_c}\right) & \textbf{\textit{if}}\ \delta \le \delta_c \\ 0 & \textbf{\textit{if}}\ \delta > \delta_c \end{cases} \tag{7}$$

Here f_t is the tensile strength and δ_c is the maximum crack opening displacement.

The crack propagates as at the crack tip the stress projection on the normal direction n of the crack to be equal to the tensile strength of the material, f_t.

3.3 Direction of crack growth

To determine the crack growth direction, we use the maximum hoop criteria. The crack is extended in the direction where the circumferential stress is maximum. Due to the plastic zone and fluctuation of numerical results in front of the crack tip the circumferential stresses on an arc of -72.5 to +72.5 degrees at a distance of 2.5 times a element characteristic length ahead the crack tip are compared to determine the crack growth direction.

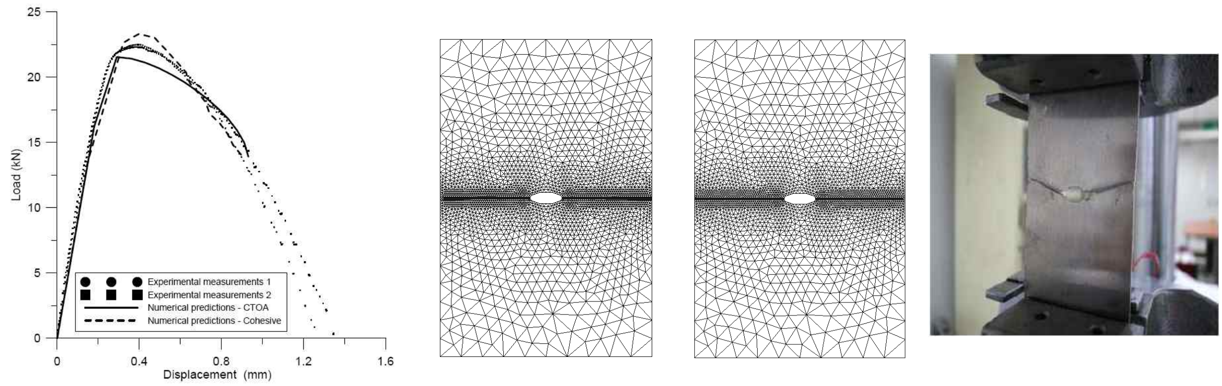
4. Numerical simulation

All numerical simulations reported here were implemented by the XFEM, which enables to propagate crack growth without remeshing. The von Mises type hardening J2-plasticity model (Simo & Hughes, 1998) and the small strain assumption were employed to capture the plastic zone in front of the crack tip. The nonlinear equilibrium equation is solved by the Newton-Raphson method combined with a line search approach. The analyses were conducted under displacement control and plane stress condition to predict the maximum load carrying capacity and a reduction in applied load required for a stable crack growth.

Test specimens, width 40mm and height 60mm, were made of 2.0mm thick 2024-T3 aluminum alloy. All specimens were initially cracked 2.0mm. The material properties were determined from a uniaxial stress-strain curve with Young's modulus $72.6\text{x}10^9$ N/m^2, Poisson's ratio 0.36, yield strength $350\text{x}10^6$ N/m^2 and linear isotropic hardening H'=$1700\text{x}10^6$ N/m^2. The specimens were applied monotonic increase of displacement.

Displacement based three-node elements were used. The criteria CTOA was 5.25 degrees measure about 1mm behind the crack tip. The linear cohesive law was used with tensile strength $460\text{x}10^6$ N/m^2 and the maximum crack opening displacement 4.45e-5 m.

Experimental (symbols) and simulated load-displacement curves (solid lines for the CTOA and dash line for the cohesive model) are shown in Fig. 1. The experimental and numerical curves agree quite well. The cohesive crack model shows very good agreement with the experimental curves. The maximum strength predicted by the cohesive crack model is large than that given by CTOA criteria. The reason for the derivation of the CTOA criteria based prediction is that in fact larger, and even much larger, CTOA was required at initiation than the value needed for stable crack growth (Scheider et al., 2006).



(a) (b) (c) (d)

Fig 1. Horizontal crack: (a) Load-displacement curve; (b) Crack trajectory by CTOA; (c) Crack trajectory by cohesive model; (d) Experiment

5. Conclusion

The elastic plastic crack growth based on the CTOA criteria or cohesive crack model showed good agreement with experimental measurements. Although the results for the CTOA model are not accurate as those for the cohesive one the former approach is numerically simple. The cohesive crack model predicts very good results and the presence of an initial crack is not necessary.

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