The Phantom-node Method for Cracked Problems in Shell Structures

지광습[*] • Chau-Dinh, Thanh[**] • 이필승[***] • 김지환[****]
Zi, Goangseup • Chau-Dinh, Thanh • Lee, Phillseung • Kim, Jihwan

Abstract

This paper presents an method, called the phantom-node method, for representing discontinuities in shell structures. By decomposing an element completely cut by a crack into two overlapped elements special treatment of the MITC3 shell element to overcome "locking phenomenon" is straightforward. Two numerical examples are provided.

keywords : phantom-node method, MITC3 shell element

1. Introduction

Shell structures are widely used in nature and in industries. In aerospace industry, the cracks are usually emanated from a row of fastener holes in a bonds, riveted lap joint in a pressurized fuselage of airplane. So that it is necessary to correctly model and predict their behavior during lifetime.

To describe the cracks in a structure, the finite element methods require the crack lines to coincide with edges of elements. It results in time-consuming remeshing as the cracks evolve. Recently, there have been such new methods as extended finite element methods, phantom-node methods or meshless methods which permit elements cut by the cracks. In this paper, the phantom-node method is applied to describe the cracks in shell structures. Without enrichments, natural assumed strain technique for removing shear locking is directly introduced with no modification.

2. The phantom-node method

Consider a domain Ω containing a crack Γ_{cr}. The domain is discretized by finite elements. Due to the crack there are elements cut by the crack. We assume here that the tip of the crack is always located on an edge of an element. So that the cracked elements are completely divided into two real sub-domains: Ω_0^+ and Ω_0^-. To obtain full interpolation bases of the real sub-domains as the standard finite approximation, the real sub-domain Ω_0^+ is extended to the phantom sub-domain Ω_P^- and vice verse. The phantom sub-domains are created by adding phantom nodes, marked by empty circles in Figure 1,

* 정회원 • 고려대학교 건축·사회환경공학과 부교수 g-zi@korea.ac.kr
** 학생회원 • 고려대학교 건축·사회환경공학과 박사과정 chdthanh@korea.ac.kr
*** 정회원 • 한국과학기술원 부교수 phillseung@kaist.edu
**** 학생회원 • 고려대학교 건축·사회환경공학과 박사과정 jihwan@korea.ac.kr

at the same position of the cracked element's nodes. As a result, the displacement approximation of an cracked element is given, Rabczuk et al. (2008),

$$\boldsymbol{u}^h(\boldsymbol{x}) = \sum_{I \in \{N_0^+, N_P^-\}} N_I(\boldsymbol{x}) H(f(\boldsymbol{x})) \boldsymbol{u}_I + \sum_{J \in \{N_0^-, N_P^+\}} N_J(\boldsymbol{x}) H(-f(\boldsymbol{x})) \boldsymbol{u}_J \tag{1}$$

Here, $f(\boldsymbol{x})$ is signed distance function measured from spatial coordinates $\boldsymbol{x}$ to the crack line; N_I and N_J are the standard shape functions; $H(x)$ is the Heaviside function; N_0^+, N_P^-, N_0^- and N_P^+ are the nodes belonging to Ω_0^+, Ω_P^-, Ω_0^-, Ω_P^+, respectively.

The displacement approximation of the cracked element actually composes of those of two regular finite elements overlapped. The displacement jump is realized by integrating over the real sub-domains, i.e. Ω_0^+ and Ω_0^-. To obtain no displacement jumps at the tip of the crack, no phantom nodes is added at nodes of the edge containing the tip.

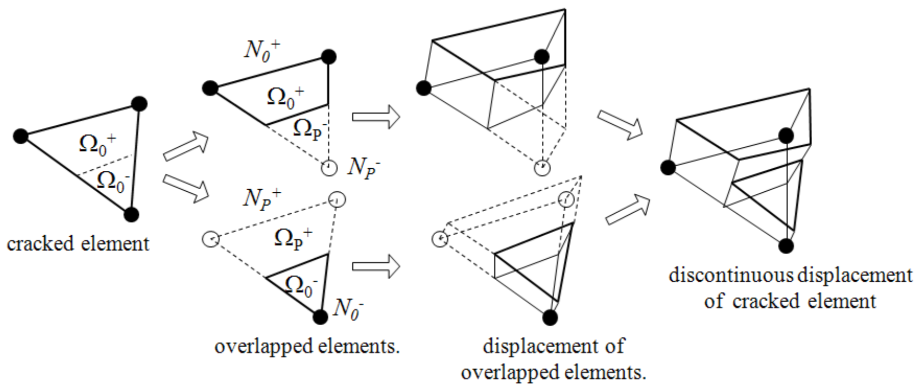

Figure 1 Description of discontinuous displacements by the phantom-node method

3. Shell elements

For elements not cut by a crack the MITC3 shell model, Lee and Bathe (2004), is employed. This shell finite element is developed from continuum mechanics displacement fields to include coupling in-plane and transverse behavior of curved structures. To remove "locking phenomenon", particularly "shear locking" in the 3-node shell element, a special assumption for transverse shear strains is introduced.

The geometry $\boldsymbol{x}$ and approximated displacement field $\boldsymbol{u}^h$ are

$$\boldsymbol{x}(r,s,t) = \sum_{I=1}^{3} N_I(r,s)\boldsymbol{x}_I + \frac{t}{2}\sum_{I=1}^{3} a_I N_I(r,s)\,\boldsymbol{V}_n^I \tag{2}$$

$$\boldsymbol{u}^h(r,s,t) = \sum_{I=1}^{3} N_I(r,s)\boldsymbol{u}_I + \frac{t}{2}\sum_{I=1}^{3} a_I N_I(r,s)\left(-\boldsymbol{V}_2^I\alpha_I + \boldsymbol{V}_1^I\beta_I\right) \tag{3}$$

Here, r, s, t is the natural coordinate system with (r,s) coinciding the mid-surface of the element; a_I is the thickness at the node I; V_n^I is the director vector; V_1^I and V_2^I are unit vectors normal to the director vector at the node I; u_I are the translation displacements; α_I and β_I are rotation of the director vector about V_1^I and V_2^I, respectively, see Figure 2.

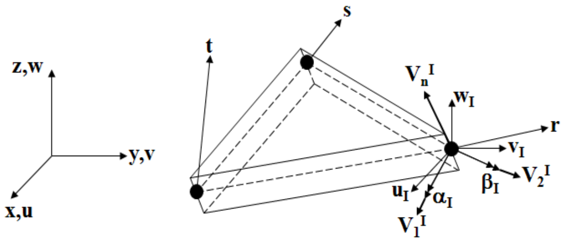

Figure 2 A 3-node shell finite element

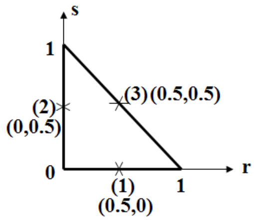

Figure 3 Tying points

Transverse shear strains in the natural coordinates are separately approximated through those derived from the displacement field (2) at tying points, lying at points as shown in Figure 3.

$$\widetilde{\epsilon_{rt}} = \epsilon_{rt}^{(1)} + cs$$
$$\widetilde{\epsilon_{st}} = \epsilon_{st}^{(2)} - cr$$
$$c = \epsilon_{st}^{(2)} - \epsilon_{rt}^{(1)} - \epsilon_{st}^{(3)} + \epsilon_{rt}^{(3)}$$

From the separately approximated transverse shear strains, the strain-displacement matrix B is modified at components related to transverse shear strains. And then the element stiffness matrix K is obtained.

For elements cut by the crack, the above procedure is computed for each overlapped element. However, numerical integration is done on the real sub-domain of each.

4. Numerical example

4.1. A cracked cylinder under pressure

A 0.1-m thick cylinder clamped at the both ends containing cracks emanated from a hole, Figure 4. is pressurized p=1MPa. Small strain and elastic analysis shown the crack opening as in Figure 5.

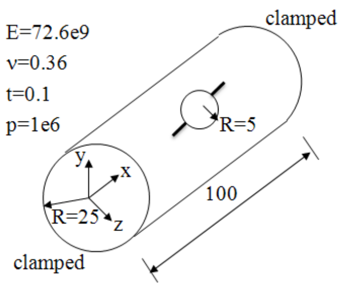

Figure 4 Geometry and material properties

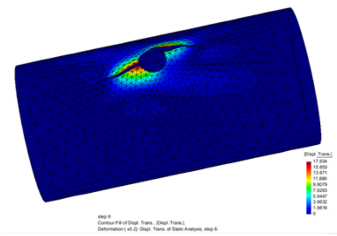

Figure 5 Displacement fileds

4.2. An edge cracked plate

Consider a fracture test of an 40x60x2-mm aluminum plate containing an initial crack a=10mm. The method is employed to simulate the stable crack growth based on the crack tip opening angle criterion during the middle-tension test. J2-plasticity model is used to capture the active plastic zone in front of the tip. The numerical load-displacement curve is good agreement with experimental measurement and the crack growth path, too.

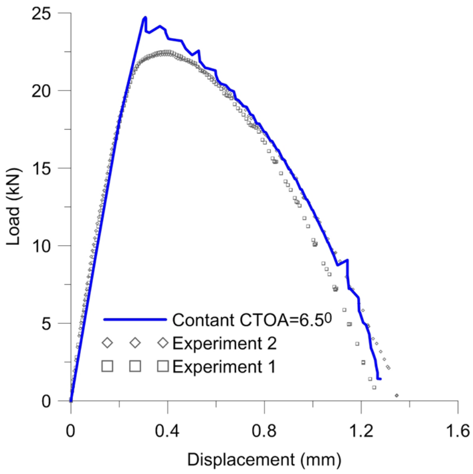

Figure 6 The load-displacement curve

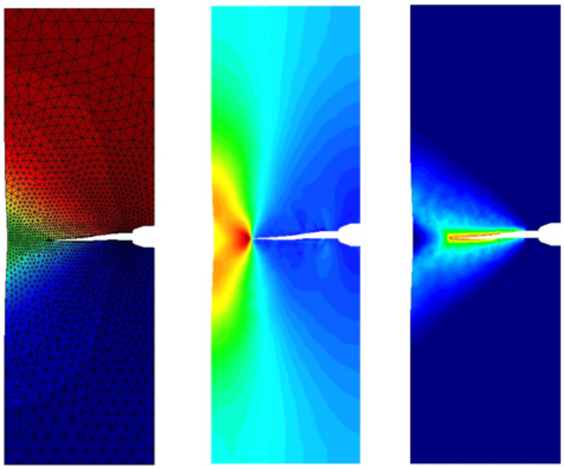

Figure 7 y-displacement; yy-stress; yy-plastic strain

4. Conclusions

The phantom-node method using the MITC3 shell element was applied to model cracks in shell structures. Thanks to absence of enrichement the approximated displacement field is continuous on each overlapped element, so that introduction of separately assumed strains is straightforward. Numerical simulation shown reasonable results.

Acknowledgements

We would like to thank the Agency for Defense Development (ADD) for their grants ADD-06-05-06, second stage.

References

Rabczuk, T., Zi, G., Gerstenberger A. and Wall W.A. (2008) A new crack tip element for the phantom-node method with arbitrary cohesive cracks, *International Journal for Numerical Methods in Engineering*, 75, pp.577~599.

Lee, P.S. and Bathe K.J. (2004) Development of MITC isotropic triangular shell finite elements, *Computer and Structures*, 82, pp.945~962.